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Rotational Splitting of Global Solar Oscillations and Its Relevance to Tests of General Relativity

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The estimation of the sun's internal angular velocity using rotational splitting of low-order, low-degree global oscillations in the limb-darkening function is reviewed. Observed spatial properties of the eigenfunctions confirm the multiplet classifications previously identified by the existence of Zeeman-like frequency patterns characteristic of rotational splitting. From the observed rotational splitting, a sidereal rotational frequency of $\gtrsim 3.0$ μHz is derived for the deep interior; this value is more than six times greater than the equatorial rotational frequency of the photosphere. Upper limits have also been estimated for the internal magnetic field by analyzing the splitting for departures from uniform spacing. The inferred angular velocity distribution, together with the estimated upper limit on the internal magnetic field, yield a gravitational quadrupole moment, J_2, of $(5.5 \pm 1.3) \times 10^{-6}$. When this result is combined with different published results from planetary radar observations, values of 0.987 ± 0.006 and 0.991 ± 0.006 are obtained for $(2+2\gamma-\beta)/3$, a combination of Eddington–Robertson PPN parameters which in general relativity takes the value of 1.

1. INTRODUCTION

Historically, one of the most important tests of theories of gravitation concerns the rates of precession of the perihelions of planetary orbits, particularly that of Mercury. In the parametrized post-Newtonian (PPN) representation of the metric, the predicted advance of the perihelion per orbital period, after correcting for the Newtonian perturbations due to the other planets, is

$$\Delta\tilde{\omega} = \frac{6\pi G M \lambda_p}{a(1-e^2)c^2} \tag{1}$$

Generally, λ_p represents

$$\lambda_p = \frac{1}{3}(2+2\gamma-\beta)+\frac{R^2c^2}{2GMa(1-e^2)}J_2 \quad (2)$$

where M and R are the mass and radius of the sun, respectively, G is the gravitational constant, a and e are the semimajor axis and eccentricity of the planetary orbit, respectively, c is the speed of light, β and γ are Eddington–Robertson parameters of the PPN formalism, and J_2 is the gravitational quadrupole moment of the sun.[1] The value of the term $(2+2\gamma-\beta)/3$ varies from one general relativistic theory to another, and its determination is one of the central objectives in planetary studies. In the general theory of relativity, for example, $(2+2\gamma-\beta)/3$ equals 1.

This now classical test of gravitational theories remains one of the more relevant tests currently available, despite the impressive results of Taylor and Weisberg (1982) on the binary pulsar PSR 1913+16. The precession of this system's periastron is $4^\circ\!.2261 \pm 0^\circ\!.0007$/year (Taylor and Weisberg, 1982), a value much larger and much more accurately known than that of Mercury's perihelion precession (~ 43″/century). Although it would appear that the periastron data imply a much stronger constraint on tests of theories of relativity, this is, unfortunately, not the case.

Taylor and Weisberg have determined the orbital details for this binary system. However, this analysis required that a theory of gravitation be assumed. In particular, masses for both objects were found by assuming Einstein's general relativistic equations of motion to hold and then using the observed gravitational red shift and the transverse Doppler shift. The time rate of change of the system's period was then calculated using general relativity, which predicts quadrupole gravitational radiation and subsequent decay. The observed change in orbital period is consistent with the calculated value. This represents an important consistency check for general relativity, even though it does not establish that the radiation is quadrupole in nature nor that a unique solution has been obtained. The results are quite impressive in light of the fact that the bimetric theories of Rosen (1974), Ni (1973), and Lightman and Lee (1973) all predict the wrong sign for the change in the orbital period.

In support of the contention that a unique solution has not been obtained, it should be noted that the theory of Moffat (1979, 1980), which is

[1]A more general expression for λ_p includes additional PPN parameters. However, they have been omitted in equation (2) because they are negligible at the level of accuracy to which $\Delta\tilde{\omega}$ for Mercury, the primary observational result considered here, is currently known. For a discussion of this point and a review of the development of the PPN formalism, see the work by Will (1980).

based on a nonsymmetric field, fits the binary pulsar data as well as does the general theory (Moffat, 1983). Moffat's theory also predicts quadrupole radiation in the decay of the orbit of the binary. It becomes apparent that an additional observable in the binary system is required to overdetermine the equations. This fact shows that the work of Taylor and Weisberg, while quite important, does not "prove" general relativity to the exclusion of all other extant theories of gravitation. As a consequence, solar system observations remain a significant source of information in testing theories of gravitation.

In planetary studies, the parameter J_2 in equation (2), normally presumed to arise from centrifugal distortion, is defined such that the sun's external gravitational potential, written in spherical polar coordinates (r, θ, ϕ) with respect to the sun's rotation axis, is

$$\phi = \frac{GM}{r}\left[1 - J_2\left(\frac{R}{r}\right)^2 P_2(\cos\theta)\right] \tag{3}$$

For Mercury, the predicted perihelion advance per unit time obtained from equation (2), in seconds of arc per century, is

$$\dot{\omega} = 42.95\left[\tfrac{1}{3}(2+2\gamma-\beta)+0.029\,J_2\times 10^5\right] \tag{4}$$

The determination of the value of $(2+2\gamma-\beta)/3$ is clearly central to a planetary test of general relativity. However, the precession of Mercury's perihelion alone cannot be used to determine $(2+2\gamma-\beta)/3$ or J_2 separately. Data on the perihelion motion of the other planets and/or other periodicities of Mercury would, in principle, provide the required independent determination of these parameters. Unfortunately, Anderson et al. (1978), in their analysis of the existing planetary data, found that the correlation between $(2+2\gamma-\beta)/3$ and J_2, after the data fit, was -0.976. This implies that these parameters are manifested almost identically in the observations. As a result, such analyses cannot lead to independent measures of these parameters with accuracies that are meaningful at this time.

The need to separate J_2 from $(2+2\gamma-\beta)/3$, in the currently available data (Anderson et al., 1977), may be lessened if it should be ascertained, by independent means, that the magnitude of J_2 is as small as that which would result if the sun rotated uniformly at the observed surface rate $(J_2 \simeq 10^{-7})$. However, the only relevant independent determinations of J_2, obtained from the solar oblateness work of the 1960s and 1970s, are one to two orders of magnitude larger than that which would result from a uniformly rotating sun. For a J_2 of this size and with this large a degree of uncertainty, further improvement in the measurement of $\dot{\omega}$ for Mercury would not lead to a more accurate value for $(2+2\gamma=\beta)/3$. Thus, our lack of a more detailed knowledge of J_2 poses a serious limitation to the present interpretation of perihelion-shift measurements. A new and more accurate measure of J_2 is obviously required in order to probe general relativistic effects via planetary observations.

The independently determined values of J_2 obtained from measures of the sun's visual oblateness (the flattening of the solar disk) have not been used for several reasons. One is the reluctance in the scientific community, the analyses of Dicke (1974) notwithstanding, to relate the visual solar oblateness to the gravitational equipotential surfaces. A second reason may be that there are discrepancies in contemporary measurements of solar oblateness. The series of measurements performed in 1966 by Dicke and Goldenberg (1974) yields

$$J_2=(24.7\pm 2.3)\times 10^{-6} \quad (5)$$

while the measurements made at SCLERA[2] in 1973 by Hill and Stebbins (1975a) yield

$$J_2=(1.0\pm 4.3)\times 10^{-6} \quad (6)$$

The interpretation of solar oblateness measurements is complicated by the well-documented existence of differences in the limb darkening between the solar poles and equator. These differences, involving the profile of the light intensity at the limb, occur on both short (~ 1 hr) and long (≥ 1 week) time scales and introduce systematic errors into measures of visual solar oblateness (Hill et al., 1974; Hill and Stebbins, 1975a, b; Hill, Stebbins and Brown, 1976). In obtaining the results given in equation (6), these limb darkening effects were measured simultaneously with the solar oblateness determination, and appropriate precautions and corrections were made. This is not the case for the results of Dicke and Goldenberg given in equation (5), and the techniques used in that measurement are vulnerable to these effects. Thus, these two determinations of the intrinsic visual solar oblateness are not equivalent in their treatment of a major systematic error, a fact which speaks directly to the existing discrepancy.

The limitations imposed by J_2 in the study of general relativistic effects have led to proposals to probe the solar gravitational field at different distances from the sun by other means. One method, mentioned previously, would compare the perihelion shifts of different planets; however, as

[2]SCLERA is an acronym for the Santa Catalina Laboratory for Experimental Relativity by Astrometry, a facility jointly operated by the University of Arizona and Wesleyan University.

Anderson et al. (1978) showed, the perihelion shifts of Venus, Earth, and Mars are not known to sufficient accuracy. Another method, also mentioned previously, would take advantage of the perturbations in Mercury's orbit induced by J_2 and by relativistic gravity. The accuracy required for the latter measurements would necessitate tracking of a spacecraft in orbit around Mercury, and preliminary studies have shown that a determination of J_2 to an accuracy of the order of 10^{-7} would be possible (Anderson et al., 1977). Another study considered the placement of a spacecraft in a highly eccentric solar orbit with a perihelion distance of four solar radii; an accuracy of 10^{-8} was projected for a measure of J_2 obtained from the study of such a spacecraft's orbit (Nordtvedt, 1977; Anderson et al., 1977). Such missions would also result in improved determinations of β and γ.

The discovery of global solar oscillations by Hill and his collaborators [for a review see the work by Hill (1978)], by Severny, Kotov, and Tsap (1976), and by Brookes, Isaak, and van der Raay (1976) has opened another route for determining J_2. The potential now exists for characterizing the deep interior of the sun more directly than was previously possible. In particular, rotational splitting of otherwise degenerate nonaxisymmetrical modes provides averages of the interior angular velocity Ω. Different modes weight Ω differently, and with a sufficient variety one can proceed toward determining the variation of Ω with position. With this new information on Ω, an improved estimate of the dynamical contribution to J_2 can be made. Bos and Hill (1983) have resolved individual eigenstates of the sun in the period range of 20 min to 2 hr. Using their data, the study of Ω is relatively convenient. It is this new approach to the determination of J_2 that we consider in the following sections.

2. ROTATIONAL SPLITTING AND THE ESTIMATION OF THE SOLAR ANGULAR VELOCITY

Slow rotation causes nonaxisymmetrical standing wave patterns to precess about the rotational axis of the sun. This is manifested as a lifting of the degeneracy in frequency with respect to the angular order m of the modes, i.e., the order of the spherical harmonic that factors from the eigenfunctions (Ledoux, 1951; Gough, 1977). We shall assume that the angular velocity $2\pi\Omega$ of the sun can be approximated by

$$\bar{\Omega} = \left[\Omega_0(r) + \Omega_2\cos^2\theta\right]\hat{k} \tag{7}$$

where $\hat{k}$ is a constant unit vector defining the axis of rotation. This form is chosen because of the θ dependence of the differential rotation observed at the surface. In this case, the frequency ν_{nlm} of a mode of radial order n and degree l, with respect to an inertial frame of reference, is approximately $\nu_{nl0} + \nu'_{nlm}$, where

$$\nu'_{nlm} = -m\int_0^R K_{nl}\left\{\Omega_0(r) + \left[\frac{(2l^2+2l-1)-2m^2}{4l^2+4l-3}\right]\Omega_2(r)\right\} dr$$

$$= -m(\Delta_{nl0} + m^3\Delta_{nl2}) \tag{8}$$

Centrifugal effects have been neglected in equation (8) as they introduce only small terms of even power in m and thus will not interfere with the determination of Δ_{nl0} and Δ_{nl2}. The functions K_{nl} are the splitting kernels for the set of modes specified by n and l, and are dependent on their eigenfunctions (Hansen, Cox, and Van Horn, 1977). The dominant term in equation (8) is linear in m and, for the modes considered here, will produce, to a good approximation, a Zeeman-like frequency pattern containing $(2l+1)$ uniformly spaced multiplet members.

Observations of different splitting frequencies Δ_{nli} provide us with differently weighted averages of $\Omega_i(r)$. The resulting sets of integral equations, generated from equation (8) for the set of Δ_{nli} specific to the observations, may then be inverted and a plausible estimate of Ω obtained. Of course, the solution cannot be unique, in part because only a finite number of averages exists.

3. PREVIOUS OBSERVATIONS OF ROTATIONAL SPLITTING

Deubner, Ulrich, and Rhodes (1979) have measured rotational splitting of p modes with high l values and 5-min periods. These oscillations only sample approximately the outer 1%, by radius, of the sun and have currently indicated only that this region rotates at about the same speed as the photosphere (Ando and Osaki, 1975). Caudell and Hill (1980) reported variations in the heights of peaks in power spectra of solar diameter measurements, which they interpreted as beats between unresolved rotationally split modes of low degree. Such modes penetrate deeply and potentially contain much diagnostic information. Nevertheless, no attempt was made to compute Ω because the aliasing that was present permitted several quite different solutions.

More recently, a splitting of roughly 0.75 μHz in 5-min dipole and quadrupole oscillations has been announced by Claverie et al. (1981). A superposed frequency analysis was performed on the power spectrum of 28 contiguous days of whole-disk Doppler data, over a frequency range that averaged 11 orders of modes of like degree. The $l = 1$ multiplets, which can be identified independently by comparing their frequencies with those obtained by Grec, Fossat, and Pomerantz (1980), were found to be split into three uniformly spaced components, and quadrupole modes into five. However, if the line shifts really represent average line-of-sight components of the oscillation velocity, as Claverie et al. presume, symmetry requires that modes with odd $(l+m)$ cannot be detected from a whole-disk integration. Thus, it appears that only $(l+1)$, and not $(2l+1)$, components ought to be detected. To account for this, Isaak (1982) has proposed the existence of a strong magnetic field in the solar core, comparable to that postulated by Dicke (1978) to explain the 12.2-day variations in the Princeton oblateness data (Dicke, 1976). Isaak contends that the wave patterns are sufficiently distorted by this field that the instrumental sensitivity to modes with odd $(l+m)$ becomes comparable to that for modes of even $(l+m)$, while the frequencies remain essentially unaltered. However, the observations of Bos and Hill (1983) reveal properties of the oscillations which are relevant to Isaak's hypothesis. These observations are discussed in the following section.

4. THE SCLERA SOLAR DIAMETER OBSERVATIONS

In 1979, oscillations were observed in functionals, δ_i, of the intensity distribution near the solar limb. Improved spatial information and signal-to-noise considerations in the observations allowed the resolution of individual normal modes in the period range 2 hr to ~ 10 min. The observational technique made possible the generation of selective power spectra for modes satisfying certain spatial symmetry relations. See the work by Bos and Hill (1983) for a more detailed description of the observational technique.

Bos and Hill found that the eigenfunctions for oscillations in this period range are, to within 20%, either symmetric or antisymmetric for reflections about the equator and disk center. Further, contiguous states of a rotationally split multiplet have opposite symmetry for reflections about the equator. This supports the use of the spherical harmonic as the factor of the eigenfunction describing the horizontal spatial properties of the oscillation.

These symmetries have in turn been used to eliminate modes from the data, simplifying the problem of mode classification. They also lend further support to the contention that the oscillations are a solar phenomenon and do not originate in the terrestrial atmosphere. Finally, the distortions in the spatial structure of the high-order modes postulated by Isaak (1982) are not consistent with these observed symmetry relations.

5. DETECTION OF ROTATIONAL SPLITTING

In view of the uniform spacing of rotationally split components theoretically predicted by Equation (8), series of uniformly spaced peaks were sought by Hill, Bos, and Goode (1982) in the power spectra. These spectra were derived from combinations of the δ_i for which the modes were symmetric or antisymmetric about the center of the disk and symmetric about the equator. Thus, only modes of even and odd degree and with even values of m were considered. Some care was taken to account for the side bands imposed by the window function, but a systematic attempt to remove them was not made. For a series of peaks to be classified as belonging to a rotationally split multiplet, the separation between peak centers was required to be uniform to within 0.07 μHz, 1/4 of the width of the peaks. Two of the multiplets identified are shown in Figure 1.

Because Hill, Bos, and Goode counted only even values of m, the frequency splitting, as viewed from the earth, was $2\Delta_{nl}$. The number of peaks in the multiplet provided a lower bound to $(l+1)$; the frequency ν_c of the central component was taken to be the frequency ν_{nl0} of the axisymmetric mode. It is always possible for $(l+1)$ to be greater than the lower bound, because modes with the highest value of m may remain unexcited. In this case, the frequency assignment for the axisymmetrical mode may differ from the correct value by an integral multiple of $2\Delta_{nl}$.

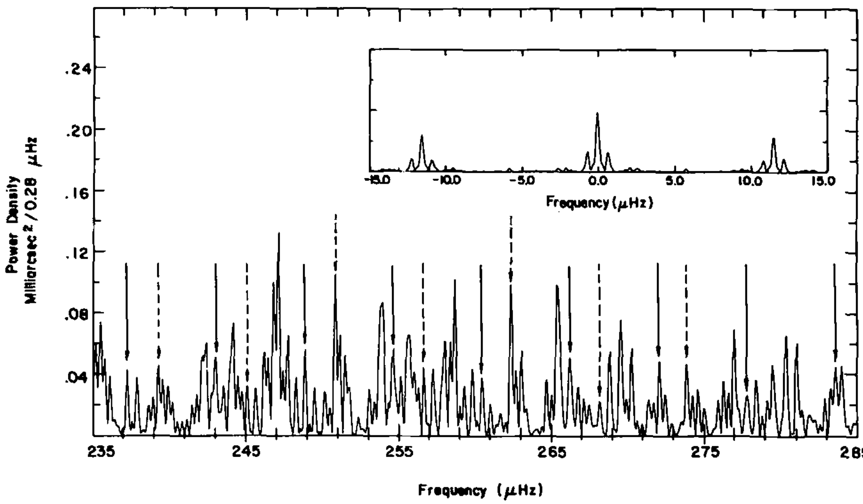

Fig. 1. Power spectrum from 235 to 285 μHz (Hill, Bos, and Goode, 1982). Dotted and solid arrows, indicate peaks comprising first two of the seven multiplets listed in Table I. Inset shows window function power spectrum.

TABLE I. Properties of Rotational Splitting[a]

Observed properties		Calculated properties		
ν_c (μHz)[b]	Δ_{nl} (μHz)	Mode used[c]	ν_c^{cal} (μHz)	Δ_{nl}^{cal} (μHz)
256.57	2.902 ± 0.008	$g_{7,6}$	253	2.89
260.41	2.934 ± 0.006	$g_{9,8}$	253	2.96
428.00	2.920 ± 0.008	$f_{0,6}$	420	2.92
540.91	1.24 ± 0.02	$p_{1,8}$	542	1.23
591.23	1.14 ± 0.02	$p_{1,10}$	586	1.15
697.96	1.40 ± 0.02	$p_{2,7}$	701	1.41
773.94	1.59 ± 0.02	$p_{3,4}$	764	1.57

[a]All frequencies given are sidereal.
[b]Error flags are ± 0.07 for all entires in this column.
[c]First subscript number refers to n, second to l.

Seven candidates for rotationally split multiplets were found. The lower bounds to l ($l_{\min}$), the frequencies ν_c, and the sidereal splittings Δ_{nl} are listed in Table I. Three of the multiplets have splittings which are very close to a period of four days. This raised the possibility that a large number of the multiplet peaks were, in fact, side bands arising from the data's 24-hr amplitude modulation and displaced 11.574 μHz on either side of a real peak (see window function in Figure 1). However, examination of the amplitude and phase of each multiplet peak, and comparison with the Fourier transform of the window function, demonstrated that all of the peaks in the multiplets were real, and did not result from aliasing.

The analysis has continued (Hill, 1984) and, to date, 28 acoustic modes and one gravity mode have been added to the list in Table I. Thus, a total of 36 multiplets have been identified. In addition, this analysis includes the odd m states, thus improving the accuracy of the splittings as well as the statistical significance of the classifications.

6. CONFIRMATION OF THE EXISTENCE OF MULTIPLETS BY OBSERVED SPATIAL PROPERTIES OF SOLAR OSCILLATIONS

Although the mode classification procedure discussed above is well supported by observational and statistical evidence, an independent means of performing the classification would be highly desirable. The observed horizontal spatial properties of the oscillations provide just such a means.

The latest results (Hill, 1984) show that these spatial properties may be used to identify the members of a given family of states, providing a method of confirming multiplet classifications based on Zeeman-like frequency patterns. By studying the real and imaginary portions of finite Fourier transforms (FFTs) of the eigenfunctions at the extreme solar limb, functional forms can be derived which establish that the index used to order a family of states in frequency can also serve to order these states in terms of spatial properties. The FFTs are obtained using the finite Fourier transform definition of the edge of the solar limb. The formalism is developed by Hill (1984). The ratio of the imaginary to real parts of the FFTs is a function of m; the functional form strongly resembles a dispersion curve for a damped system with several natural frequencies. The preliminary results of this work are shown in Figure 2. The fact that the states can be ordered according to these spatial properties shows that identifiable multiplets do exist. The properties shown in Figure 2 can also be used to identify, with minimal ambiguity, the $m = 0$ state, a process which is pertinent to studies of static properties of the solar interior and to tests of various gravitational theories.

Preliminary results obtained using this technique confirm multiplet classifications based on Zeeman-like patterns. These spatial properties provide further support for the model of the sun as an axially symmetric system with the internal axis of symmetry aligned with that defined by the surface rotation. The use of the spherical harmonic to represent the horizontal portion of the eigenfunction is also confirmed.

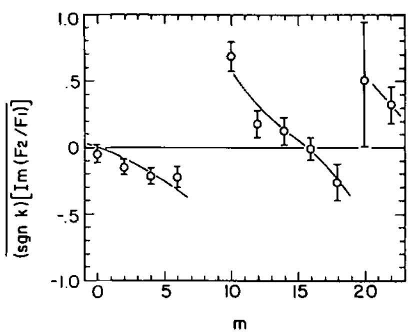

Fig. 2. The ratio of the imaginary to real parts of, essentially, the finite Fourier transform of the eigenfunction at the solar limb. The statistically meaningful deviation from zero of this ratio confirms the classification of the multiplets previously based on the presence of Zeeman-like frequency patterns.

7. UPPER LIMIT ON INTERNAL DIFFERENTIAL ROTATION

Differential rotation at the surface is quite large, with

$$\Omega(r_0,\theta) = (0.451 - 0.129\cos^2\theta)\ \mu\text{Hz} \tag{9}$$

(Allen, 1973). This type of deviation from uniform rotation on spheres produces departures from the equally spaced frequency pattern in a given multiplet as seen in equation (8).

The $P_{1,22}$ multiplet has been identified using both the existence of a Zeeman-like frequency pattern and the observed $e^{im\phi}$ dependence discussed in the previous section. This multiplet contains $(2l+1)$ states, i.e., 45 and 30 states have been observed. It was chosen to be examined for evidence of differential rotation because of the large number of multiplet members observed and because, of the 19 acoustic modes with $n=1$ that have thus far been classified, its eigenfunction is the least penetrating. In the associated splitting kernel, the second of the two peaks, which is the one located at the largest radius, occurs at approximately $0.987R$.

The observed m^3 dependence for this multiplet has the coefficient

$$\Delta_{1,22,2} = (-0.6 \pm 0.6) \times 10^{-5}\ \mu\text{Hz} \tag{10}$$

This corresponds to an average Ω_2 of

$$\langle \Omega_2 \rangle = (-6 \pm 6)\ \text{nHz} \tag{11}$$

or

$$\frac{\langle \Omega_2 \rangle}{\Omega_2(R)} = 0.045 \pm 0.045 \tag{12}$$

where the average is defined by equation (8).

Assuming that $|\Omega_2|$ is a monotonically decreasing function for decreasing r, we can conclude that, at a depth of 1.3% of the solar radius, Ω_2 is $(18 \pm 18)\%$ of the surface differential rotation, to a first approximation. The differential rotation is thus confined to a fraction of a percent of the solar radius, and is clearly a shallow surface phenomenon. This very small upper limit on the effects of differential rotation shows that the assumption of uniform rotation on spheres made by Hill, Bos, and Goode (1982) was in fact appropriate. Further analysis for rotational splitting can now proceed, to a good approximation, under the same assumption.

8. ANALYSIS OF THE DATA AND DETERMINATION OF THE SOLAR ANGULAR VELOCITY

In order to identify the modes responsible for these data, Hill, Bos, and Goode (1982) compared the ν_c with the frequencies of the axisymmetric modes of the solar model computed by Saio (1981). The modes selected in this analysis ($l = l_{min}$ in all cases), as well as their calculated frequencies, are given in Table I. The calculated frequencies are in reasonable agreement with the observed ones. The solar model of Saio is a standard one with abundances $X = 0.74$, $Y = 0.24$, and $Z = 0.02$, where X, Y, and Z represent hydrogen, helium, and heavy metals, respectively. The model also has a depth for the convection zone of 25% of the solar radius. The splitting kernels, K_{nl}, some of which are illustrated in Figure 3, were then constructed.

The splittings of the seven multiplets were fitted by a rotation curve calculated using a constrained least-squares analysis; the curve was constrained to be monotonic (as implied by general stellar evolution considerations) and constrained to the observed equatorial value of 0.456 μHz at the surface. Figure 4 shows the resulting rotation curve; its segmented structure reflects the type of theoretical curve used in the least-squares analysis. The predicted splittings are quite consistent with the observed splittings (see Table I).

The dynamical unitless quadrupole moment due to centrifugal stretching, $J_{2,\Omega}$, was obtained by Hill, Bos, and Goode (1982) as a numerical

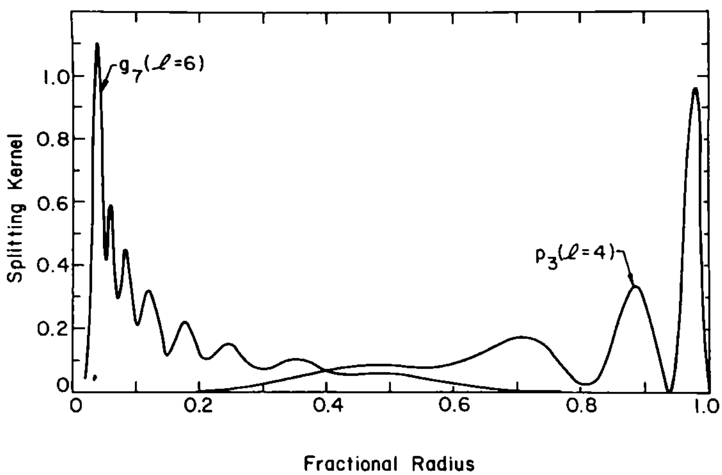

Fig. 3. Splitting kernels, K_{nl}, for two different multiplets (Hill, Bos, and Goode, 1982).

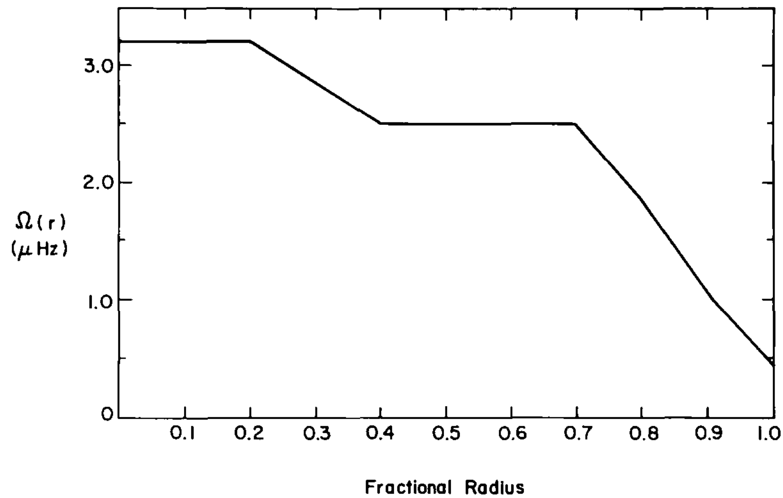

Fig. 4. Inferred $\Omega(r)$ (Hill, Bos, and Goode, 1982).

solution of the standard Goldreich–Schubert (1968) equation, yielding

$$J_{2,\Omega} = 5.5 \times 10^{-6} \tag{13}$$

9. LIMITS ON THE SUN'S CORE MAGNETISM OBTAINED FROM SOLAR OSCILLATIONS

Many interior properties of the sun are accurately known; however, the magnetic field of the solar core remains the object of considerable speculation. Hill, Bos, and Goode (1982) argued that these magnetic fields are sufficiently weak that they make a negligible contribution to the sun's quadrupole moment. On the other hand, Dicke (1976) suggested that a significant portion of the visual solar oblateness obtained by Dicke and Goldenberg (1974) is caused by strong magnetic field effects, and proposed an oblique magnetic rotator model containing magnetic fields on the order of 10^8 G. Ulrich and Rhodes (1982) stated, with some reservations, that a poloidal field of 3×10^8 G could account for certain properties of the 5-min period oscillation data.

Magnetic fields, like rotation, produce a fine structure in solar oscillations. Provided the fields are sufficiently intense, their effects should be detectable at the current level of observational accuracy. Dziembowski and Goode (1983) used the analysis of oscillation data by Hill, Bos, and Goode

(1982) to calculate limits on internal poloidal and toroidal magnetic fields, and thus on the part of the quadrupole moment of the sun due to magnetism. They found the upper limit for the poloidal field to be

$$B_p < 1 \text{ MG} \tag{14}$$

Similarly, by assigning all of the observational error to a toroidal field, they obtained its upper limit:

$$B_T < 3 \text{ MG} \tag{15}$$

These limits allow corresponding limits to be placed on their contributions to J_2; these can be written, in megagauss, as

$$J_{2,B} \leqslant 0.2(B_P)^2 \times 10^{-6}$$

and

$$J_{2,B} \leqslant 0.5(B_T)^2 \times 10^{-7} \tag{16}$$

for the poloidal and toroidal fields respectively. Thus, the preliminary results of Dziembowski and Goode (1983), shown in equations (14) and (15), indicate that magnetic fields make a negligible contribution to the quadrupole moment of the sun relative to the dynamical part as found by Hill, Bos, and Goode (1982).

10. CONCLUSION

The properties of the splitting of modes of oscillation have been used to obtain an estimate of the dynamical and magnetic contributions to the gravitational quadrupole moment of the sun. This estimate is

$$J_2 = (5.5 \pm 1.3) \times 10^{-6} \tag{17}$$

A very important feature of this work is that the derived value of J_2 is more sensitive to the properties of the observed multiplets and their associated splittings than to the inferred variation of the internal rotation with radius. This feature allowed the calculation of J_2 with a relatively small error, $\pm 1.3 \times 10^{-6}$, even though $\Omega(r)$ is not as accurately determined. Furthermore, the values of both $\Omega(r)$ and J_2 are reasonably insensitive to the details of the standard solar model used.

The value reported by Hill, Bos, and Goode for J_2 is the same as that obtained solely from the dynamical contribution; the effects of internal magnetic fields were considered and found to be negligible at the current level of accuracy for the larger dynamical contribution.

The implications of this estimate of J_2 for planetary tests of gravitation theories can be assessed using radar distance measurements to Mercury. For the data of Shapiro et al. (1976), Hill, Bos, and Goode found

$$\tfrac{1}{3}(2+2\gamma-\beta)=0.987\pm0.006 \tag{18}$$

When the data of Anderson et al. (1978) were used, they obtained

$$\tfrac{1}{3}(2+2\gamma-\beta)=0.991\pm0.006 \tag{19}$$

The error of 0.006 in these two results arises equally from errors in J_2 and $\dot{\omega}$.

Independent analyses of the splittings reported by Hill, Bos, and Goode have been performed. Gough (1982) published results for J_2 which differed from those obtained by Hill, Bos, and Goode. Although both works utilized data obtained at SCLERA, Gough's results were based upon an earlier phase of data analysis. The data were subsequently subjected to a more complete analysis, from which the value for J_2 given by Hill, Bos and Goode was derived. Campbell et al. (1983) have also performed an extensive analysis of these same data, and obtained a result for J_2 similar to that of Hill, Bos, and Goode.

The analysis of the full set of 36 multiplets has not yet been completed. However, the preliminary indications are consistent with the conclusions of Hill, Bos, and Goode (1982), and it is expected that the quantity and quality of the enlarged set will lead to even tighter constraints on J_2. The splitting of the multiplets appears to be a sensitive indicator of both rotational and magnetic contributions to J_2, and the prognosis is good for continued improvements in observations of solar oscillations. For these reasons, it is projected that the accuracy to which J_2 is known will improve considerably during this decade, keeping pace with improvements anticipated in planetary observations. Determination of the value of J_2 may no longer place limitations upon planetary tests of general relativistic theories as it has in the past.

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